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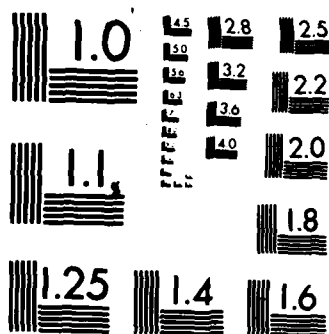
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LINEAR OR NONLINEAR WAVE PROBLEMS WITH INPUT SETS

FINAL REPORT

21 DECEMBER 1983

U. S. ARMY RESEARCH OFFICE

CONTRACT NO. DAAG 29-81-K-0042

SCHOOL OF MATHEMATICS  
GEORGIA INSTITUTE OF TECHNOLOGY  
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. DAAG 137 811	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) LINEAR OR NONLINEAR WAVE PROBLEMS WITH INPUT SETS		5. TYPE OF REPORT & PERIOD COVERED Final Report 310283 6/14/81 - 11/30/83
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) W. F. Ames		8. CONTRACT OR GRANT NUMBER(s) DAAG 29-81-K-0042
9. PERFORMING ORGANIZATION NAME AND ADDRESS School of Mathematics Georgia Institute of Technology Atlanta, GA 30332		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE 21 December 1983
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)  N/A		
18. SUPPLEMENTARY NOTES The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Nonlinear hyperbolic equations, group analysis, input data sets, numerical enclosure of solutions, nonlinear parabolic equations, Navier-Stokes equations, von Karman equations.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report presents a review of results and a list of publications developed in connection with ARO Contract No. DAAG 29-81-K-0042 for the period June 14, 1981 to November 30, 1983. Results presented concern the group analysis of hyperbolic and parabolic systems, their types of solutions and numerical enclosure of their solutions when input data sets are present.		

## 1. General Discussion of Research

This research project is concerned with analytical and numerical studies of linear and nonlinear wave problems with input data sets. On the analytical side, we were concerned with the examination of a variety of analytic techniques and applying them to nonlinear wave problems. The most general of those methods, that of group analysis, was applied with much success as will be seen in the List of Publications (Paragraph 3).

Group theoretic methods were found useful in the study of various nonlinear equations. In particular, they permit us to (i) predict the forms of various types of solutions which the system will admit; (ii) sometimes obtain exact solutions useful in ascertaining whether numerical algorithms give required accuracy (by comparison); (iii) provide a systematic way of improving numerical algorithms; (iv) permit the conversion of difficult boundary value problems to initial value problems (which are easier to solve numerically); (v) provide a systematic way of deciding whether a system has a "similar" solution; (vi) provide a systematic way of ascertaining whether an approximate method is compatible with some underlying theory (such as Newtonian Mechanics); (vii) providing assistance in sensitivity analysis, via a reduction in dimension.

Indeed, group theoretic methods seem to be the only analytic methods applicable to both linear and nonlinear problems.

On the numerical side, our goal was the development of a constructive, easily computed enclosure ("good" upper and lower bounds) of the set of solutions (depending on input parameters varying in intervals) for nonlinear wave problems. In many physical problems, the parameters (density, modulus of elasticity viscosity) are poorly known and can only be specified to lie in some intervals. Rather than solving a large number of problems with specified parameter values, we suggest that an enclosure of the solutions is preferable for the purposes of the design engineer, for example. We have been successful in developing such

an approach for the class of problems  $u_{tt} + cu_t - f(u_x) u_{xx} = 0$  as will be discussed in Paragraph 2. A paper describing the method and numerical calculations will be included in the final report.

Applications to other more general classes of nonlinear hyperbolic or parabolic problems is straightforward provided there are only input sets of constant parameters. Input sets of (continuous) functions are not permitted since sample problems from an earlier paper (Adams, E. and Ames, W.F., "Linear or nonlinear hyperbolic wave problems with input sets (Part I)", J1. Engineering Math. 16 (1982), 23-45) revealed that this may give rise to instabilities of the resonance or parameter resonance type, unless there is sufficiently strong dissipation (damping) in the equations.

The method makes use of (i) a Fourier-polynomial whose time-dependent coefficients are such that the partial differential equation is satisfied on grid lines  $x = x_1$  and, (ii) an interval extension of the first order remainder term in a Taylor expansion of the set of solutions in terms of the initial parameters and equation parameters. Using (i), the problem is approximated by a vector nonlinear ordinary ivp (initial value problem) with initial and parameter intervals. A Taylor-approximation of a set of solutions may be inaccurate for the case of sufficiently large time  $t$ . Therefore, here the Taylor-representation is computed only for an interval  $[0, t^*]$  comparable with one timewise wave length of the (almost) periodic solutions of the ibvp. Then, it follows from the autonomous character of the ibvp that a "mapping into themselves" of the input intervals at time  $t^*$  establishes the confinement of the set of values, of the set of solutions, to uniformly bounded and known intervals for all  $t \geq 0$ .

The set of solutions to be enclosed arises from perturbations in a neighborhood of the trivial\* solution of the ibvp, i.e., the analysis provides a quantitative and practical stability analysis for the trivial solution in a finite

\*Neighborhoods of nontrivial periodic or steady-state solutions can also be investigated.

initial and parameter neighborhood. In a "practical stability analysis", perturbations of both initial data and coefficients are admitted.

## 2. Summary of the Most Important Results

a) Paper A-1 was motivated by a physical problem in which the constitutive law has (natural) creep properties in one domain of the stress-strain plane and relaxation properties in another domain. General constitutive laws remaining invariant under the dilatation and spiral groups are determined. Then exact invariant solutions are constructed for a classical (linear) material, for a quadratic constitutive law and for a linear viscoelastic material. Lastly, a viscoplastic material is studied in the light of group analysis and the machinery to determine the solution (numerically) for a variety of viscoplastic problems is established.

b) Papers A-2 and B-1 form a complete picture of the possible invariant solutions for the celebrated von Karman equations for the large deflection of elastic plates. The full symmetry group for the steady equations is a finitely generated Lie group with ten parameters. For the time dependent system the full symmetry group is an infinite parameter Lie group. Several subgroups of the full group are used to generate exact solutions. This system of equations is shown to be consistent with Newtonian Mechanics. In a related paper, B-3, questions are raised about the consistency with Newtonian Mechanics of the Tadjbakhsh-Saibel generalization of these equations. Of considerable mathematical interest is the idea of raising the order to integrate certain nonlinear second, third and higher order equations. The contents of these papers was reported at the First Army Conference on Applied Mathematics and Computing held at George Washington University (1983).

c) Papers A-3 were reported at the Tenth World Congress of IMACS (International Association for Mathematics and Computers in Simulation) held in Montreal, August, 1982. A preliminary report was also given to the Chemical Engineering World Congress (Montreal 1981) in an invited paper.

Using the machinery of Lie theory (groups and algebras) applied to the Navier-Stokes equations a number of exact solutions for the steady state are derived in (two) three dimensions. It is then shown how each of these generates an infinite number of time dependent solutions via (three) four arbitrary functions of time. This algebraic structure also provides the mechanism to search for other solutions since its character is inferred from the basic equations.

d) Paper A-4 attacks a problem related to part c) above. Nonlinear models of pollutant transport in air and water leads to vector semi-linear diffusion equations of the form  $u_t - u_{xx} = f(u)$ , where  $f(u)$  is a nonlinear (vector) function from  $R^n$  to  $R^n$ . The goal of the paper is to find conditions on  $f(u)$ , the so-called classification problem, for which invariant solutions exist.

e) Paper B-2 constitutes preliminary work to a PhD Dissertation by V.J. Ervin. It was presented in an invited paper to Wave Phenomena 83, a conference held at the University of Toronto in June, 1983.

A gas dynamic model of the pellet fusion process having a time-invariant source term is studied by means of group analyses. Some exact solutions of this nonlinear system are constructed for specific (physical) values of parameters. The development of multiple shock waves is demonstrated in several cases analytically.

f) Paper C-1 introduces a generalized idea of separation of variables, called implicit separation, of which simple separation is a special case. Equations of the form  $u_{xt} = f(u)$  are solved.

g) Paper C-2 forms the heart of the numerical method described in the first section. A nonlinear autonomous dissipative hyperbolic initial boundary value problem with a parameter  $B$  given on an interval  $B \in [B, \bar{B}]$  is studied and initial data on intervals is also admitted. The algorithm for determining the enclosure of the set of values of classical solutions is obtained by using a Fourier polynomial with time dependent coefficients and an interval extension of the remainder term in a Taylor representation of the set of solutions in terms of the



initial parameters and equation parameter. The analysis provides an enclosure and a quantitative and easily computed stability analysis for the trivial solution. It is also demonstrated that standard finite difference methods cannot be employed because of local discretization errors.

h) Paper D-1 concerns wave propagation in reacting gases with input sets.

An estimation (upper and lower bounds) of the solution, for all admitted values of the parameters (input sets) of  $u_t + (1+au^p) u_x = b u^{n+1}$ ,  $u(x,0)=g(x)$ ,  $u_x(0,t)=0$  has been developed. The computations, having been delayed, are nearing completion.

### 3. List of Publications

#### A. Published Papers

1. Ames, W. F. and Suliciu, I., "Some exact solutions for wave propagation in viscoelastic, viscoplastic materials and electrical transmission lines", Int. Jl. Nonlinear Mechanics 17 (1982), 223-230.
2. Ames, K. A. and Ames, W. F., "On Group Analysis of the von Karman equations", Nonlinear Analysis: TMA, 8 (1982), 845-853.
3. Boisvert, R.E., Ames, W.F. and Srivastava, U.N., "Solutions of the Navier-Stokes equations by group methods" Proceedings IMACS 10th World Congress, Montreal, Canada (August 8-13, 1982) Vol. 3, 31-35  
"Group properties and new solutions of the Navier-Stokes equations", Jl. Engineering Mathematics, 17 (1983), 203-221.
4. Zulehner, W. and Ames, W.F., "Group Analysis of a semilinear vector diffusion equation", Nonlinear Analysis: TMA, 7 (1983), 945-969.

#### B. Papers in Press

1. Ames, K.A. and Ames, W.F., "Analysis of the von Karman equations by group methods", Proceedings of the First Army Conference on Applied Mathematics and Computing (George Washington University 1983)
2. Ervin, V.J., Ames, W.F. and Adams, E., "Nonlinear waves in the pellet fusion process", WAVE Phenomena 83 (Proceedings of a conference at the University of Toronto, 1983) North Holland Publishing Company.
3. Ames, W. F. "Invariant solutions for large deflection of elastic plates", Saibel Anniversary Volume (1984).

C. Papers Submitted

1. Ames, W.F. and Adams, E. "Implicit Separation of semi-linear wave equations".
2. Adams, E., Lohner, R., and Ames, W.F., "Linear or nonlinear hyperbolic wave problems with input sets II".

D. Papers in Preparation

1. Cordes, D., Ames, W.F. and Adams, E., "Wave propagation in reacting gases".

4. List of Participating Scientific Personnel

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D. Cordes*	Graduate Student	University of Karlsruhe
V. Ervin*	Graduate Student	Georgia Institute of Technology
M. Kvale*	Senior	Georgia Institute of Technology
R. Lang	Graduate Student	Georgia Institute of Technology
R. Lohner	Graduate Student	University of Karlsruhe
W. Zulehner*	Faculty Colleague	Georgia Institute of Technology

\*Supported by ARO Funds

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